Multi Criteria Decision Making in Fuzzy Description Logics: A First Step

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Introduction

- In the last years the interest in ontologies has significantly grown
- An ontology is defined as an explicit and formal specification of a shared conceptualization
- Description Logics (DLs) are a family of logics that are the logical foundation of the standard W3C ontology language OWL [HPS04].

- It is widely agreed that "classical" ontology languages are not appropriate to deal with fuzzy/vague knowledge
- Fuzzy ontologies emerge as useful in several applications, such as multimedia information retrieval, image interpretation, ontology mapping, matchmaking and the Semantic Web [LS08]
- Several fuzzy extensions of DLs can be found in the literature (see the survey in [LS08])
- Some fuzzy DL reasoners have been implemented, such as FUZZYDL [BS08], DELOREAN [BDGR08] or FIRE [SSSK06].

- In this work, we make a first step in combining Multi-Criteria Decision Making (MCDM) and fuzzy DLs
 - ightarrow ightarrow fuzzy knowledge assisted approach to decision making

Preliminaries: Mathematical Fuzzy Logic [Háj98]

- Fuzzy statements: $\langle \phi, n \rangle$, where $n \in [0, 1]$ and ϕ is a statement
 - The degree of truth of ϕ is at least n
- Fuzzy interpretation: I : Atoms → [0, 1] and is then extended inductively:

$$\begin{split} \mathcal{I}(\phi \wedge \psi) &= \mathcal{I}(\phi) \otimes \mathcal{I}(\psi) & \mathcal{I}(\phi \vee \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi), \\ \mathcal{I}(\phi \to \psi) &= \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi) & \mathcal{I}(\neg \phi) = \ominus \mathcal{I}(\phi), \\ \mathcal{I}(\exists x.\phi(x)) &= \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c)) & \mathcal{I}(\forall x.\phi(x)) = \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c)) \end{split}$$

 $\otimes, \oplus, \Rightarrow$, and \ominus are truth combination functions

	Łukasiewicz Logic	Gödel Logic	Product Logic	"Zadeh Logic"
a⊗b	max(a + b - 1, 0)	min(<i>a</i> , <i>b</i>)	a · b	min(<i>a</i> , <i>b</i>)
$a \oplus b$	min(<i>a</i> + <i>b</i> , 1)	_ max(<i>a</i> , <i>b</i>)	$a + b - a \cdot b$	max(<i>a</i> , <i>b</i>)
$a \Rightarrow b$	$\min(1-a+b,1)$	$\begin{cases} 1 & \text{if } a \leqslant b \\ b & \text{otherwise} \end{cases}$	min(1, <i>b/a</i>)	max(1 - a, b)
⊖ a	1 – <i>a</i>	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	1 – <i>a</i>

- $\blacktriangleright \mathcal{I} \models \langle \phi, n \rangle \text{ iff } \mathcal{I}(\phi) \ge n$
- Best Entailment Degree (BED): $bed(\mathcal{K}, \phi) = \sup \{r \mid \mathcal{K} \models \langle \phi, r \rangle \}$
- ▶ BED can be computed as (where $\phi \leq x$ is $\langle \neg \phi, 1 x \rangle$)

 $bed(\mathcal{K}, \phi) = \min x$. such that $\mathcal{K} \cup \{\phi \leq x\}$ satisfiable

E.g., for Łukasiewicz logic, we may use Mixed Integer Linear Programming

$$bed(\mathcal{K}, \phi) = \min x. \text{ such that}$$

$$x \in [0, 1], x_{\neg \phi} \ge 1 - x, \sigma(\neg \phi),$$
for all $\langle \phi', n \rangle \in \mathcal{K}, x_{\phi'} \ge n, \sigma(\phi'),$

$$\sigma(\phi) = \begin{cases}
x_{p} \in [0, 1] & \text{if } \phi = p \\
x_{\phi'} = \ominus x_{\phi}, x_{\phi} \in [0, 1] & \text{if } \phi = \neg \phi' \\
x_{\phi_1} \otimes x_{\phi_2} = x_{\phi}, \\
\sigma(\phi_1), \sigma(\phi_2), x_{\phi} \in [0, 1] & \text{if } \phi = \phi_1 \land \phi_2 \\
x_{\phi_1} \oplus x_{\phi_2} = x_{\phi} & \text{if } \phi = \phi_1 \lor \phi_2 \\
\sigma(\neg \phi_1 \lor \phi_2) & \text{if } \phi = \phi_1 \to \phi_2.
\end{cases}$$

Preliminaries: MCDM Basics

- Alternatives A_i: different choices of action available to the decision maker to be ranked
- Decision criteria C_j: different dimensions from which the alternatives can be viewed and evaluated
- Decision weights w_j: importance of a criteria
- Performance weights a_{ij}: performance of alternative w.r.t. a decision criteria

			Criteria					
		w ₁	W2	•	•	wm		
Alternatives		<i>C</i> ₁	C ₂	•	•	Cm		
<i>x</i> ₁	A ₁	a ₁₁	a ₁₂	•	•	a _{1m}		
<i>x</i> ₂	A2	a ₂₁	a ₂₂	•	•	a _{2m}		
·	•	•	•	•	•	•		
•	· ·	•	•	•	•	•		
хn	An	a _{n1}	a _{n2}	•		anm		

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Final ranking value x_i:

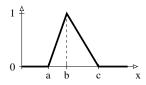
$$x_i = \sum_{j=1}^m a_{ij} w_j$$

Optimal alternative A*:

$$A^* = \arg \max_{A_i} x_i$$

Preliminaries: Fuzzy MCDM Basics

- Principal difference: weights w_i and performance a_{ij} are fuzzy numbers
- Fuzzy number ñ: fuzzy set over relas with triangular membership function *tri*(*a*, *b*, *c*). Intended being an approximation of the number *b*



- Any real value *n* is seen as the fuzzy number tri(n, n, n)
- ► Arithmetic operators +, -, · and ÷ are extended to fuzzy numbers
 - For $* \in \{+, \cdot\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * a_2, b_1 * b_2, c_1 * c_2)$
 - For $* \in \{-, \div\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * c_2, b_1 * b_2, c_1 * a_2)$

Final ranking value x_i: fuzzy number

$$ilde{x}_i = \sum_{j=1}^m ilde{a}_{ij} \cdot ilde{w}_j$$

Optimal alternative A*:

$$A^* = \arg \max_{A_i} x_i^{defuzzy}$$

using some defuzzification method for fuzzy numbers and the second second

Towards MCDM in Fuzzy Description Logic

- Our extension of to fuzzy DLs is grounded on the fuzzy DL ALCF(D) [Str05]
- We will just provide a minimal variant of ALCF(D) to deal with MCDM
- Recall that fuzzy ALCF(D) is the basic DL ALC extended with functional roles (letter F) and concrete domains [LM07] (letter D) allowing to deal with data types such as strings, integers, reals and fuzzy membership functions

Description Logics (DLs)

- The logics behind OWL-DL and OWL-Lite, http://dl.kr.org/.
- Concept/Class: names are equivalent to unary predicates
 - In general, concepts equiv to formulae with one free variable
- Role or attribute: names are equivalent to binary predicates
 - In general, roles equiv to formulae with two free variables
- Taxonomy: Concept and role hierarchies can be expressed
- Individual: names are equivalent to constants
- Operators: restricted so that:
 - Language is decidable and, if possible, of low complexity
 - No need for explicit use of variables
 - ▶ Restricted form of \exists and \forall
 - Features such as counting can be succinctly expressed

The Crisp DL Family

- A given DL is defined by set of concept and role forming operators
- Basic language: ALC(Attributive Language with Complement)

Syntax	Semantics	Example
$C, D \rightarrow \top$	$ \top(x)$	
1	$\perp (x)$	
A	A(x)	Human
$C \sqcap D$	$C(x) \wedge D(x)$	Human ⊓ Male
$C \sqcup D$	$C(x) \vee D(x)$	Nice 🗆 Rich
$\neg C$	$ \neg C(x)$	¬Meat
∃ <i>R</i> . <i>C</i>	$\exists y.R(x,y) \wedge C(y)$	∃has_child.Blond
∀ <i>R</i> . <i>C</i>	$\forall y.R(x,y) \rightarrow C(y)$	∀has_child.Human
$C \sqsubseteq D$	$\forall x. C(x) \to D(x)$	Happy_Father \sqsubseteq Man $\sqcap \exists$ has_child.Female
a:C	<i>C</i> (<i>a</i>)	John:Happy_Father

Example: GIS Quality Assessment Ontology [OWML08]

Sogontology.owl (http://www.owl-ontologies.com/osqontology.owl)							₿8 (88 (Q	
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Note on DL Naming

- $\mathcal{AL}: \quad C, D \quad \longrightarrow \quad \top \mid \perp \mid A \mid C \sqcap D \mid \neg A \mid \exists R. \top \mid \forall R. C$
 - C: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + C$
 - S: Used for ALC with transitive roles R_+
 - \mathcal{U} : Concept disjunction, $C_1 \sqcup C_2$
 - \mathcal{E} : Existential quantification, $\exists R.C$
 - \mathcal{H} : Role inclusion axioms, $R_1 \sqsubset R_2$, e.g., is component of \Box is part of
 - \mathcal{N} : Number restrictions, ($\geq n R$) and ($\leq n R$), e.g., ($\geq 3 has_Child$) (has at least 3 children)
 - Q: Qualified number restrictions, ($\ge n R.C$) and ($\le n R.C$), e.g., $(\leq 2 has Child.Adult)$ (has at most 2 adult children)
 - \mathcal{O} : Nominals (singleton class), {*a*}, *e.g.*, \exists has child.{mary}. **Note**: *a*: *C* equiv to $\{a\} \sqsubseteq C$ and (a, b): *R* equiv to $\{a\} \sqsubseteq \exists R.\{b\}$
 - \mathcal{I} : Inverse role, R^- , e.g., isPartOf = hasPart⁻
 - \mathcal{F} : Functional role, f, e.g., functional(hasAge)
- \mathcal{R}_+ : transitive role, e.g., transitive(isPartOf)
 - \mathcal{R} : role inclusions with composition, $R_1 \circ R_2 \sqsubseteq S$, *e.g.*, *isPartOf* ∘ *isPartOf* ⊂ *isPartOf*

For instance,

 $SHIF = S + H + I + F = ALCR_+HIF$ **OWL-Lite** $SHOIN = S + H + O + I + N = ALCR_+HOIN$ OWL-DL $SROIQ = S + R + O + I + Q = ALCR_+ROIN$ OWI 2 ◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

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Fuzzy DLs Basics

The semantics is an immediate consequence of applying mathematical fuzzy logic to the First-Order-Logic translation of DLs expressions

Interpretation:	$ \begin{array}{lll} \mathcal{I} & = & \Delta^{\mathcal{I}} \\ \mathcal{C}^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \to [0, 1] \\ \mathcal{R}^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \end{array} $	$\begin{matrix} \otimes \\ \oplus \\ \ominus \\ \rightarrow \\ [0,1] \end{matrix} \qquad \begin{array}{c} \otimes \\ \oplus \\ \ominus \\ \Rightarrow \\ \end{array}$	= = =	t-norm s-norm negation implication
	Syntax	Semantics		
Concepts:	$\begin{array}{cccc} C,D & \longrightarrow & \top \mid & & \\ & & \bot \mid & & \\ & & C \sqcap D \mid & \\ & C \sqcup D \mid & \\ & & C \sqcup D \mid & \\ & \neg C \mid & \\ & \exists R.C \mid & \\ & \forall R.C & \end{array}$	$ \begin{array}{c} \top^{\mathcal{I}}(x) \\ \perp^{\mathcal{I}}(x) \\ A^{\mathcal{I}}(x) \\ (C_1 \sqcap C_2)^{\mathcal{I}}(x) \\ (C_1 \sqcup C_2)^{\mathcal{I}}(x) \\ (\neg C)^{\mathcal{I}}(x) \\ (\exists R.C)^{\mathcal{I}}(x) \\ (\forall R.C)^{\mathcal{I}}(u) \end{array} $	= E = = =	$ \begin{array}{l} 1 \\ 0 \\ [0,1] \\ C_1^{\ T}(x) \otimes C_2^{\ T}(x) \\ C_1^{\ T}(x) \oplus C_2^{\ T}(x) \\ \oplus C^{\ T}(x) \\ \sup_{y \in \Delta^{\mathcal{I}}} R^{\ T}(x,y) \otimes C^{\ T}(y) \\ \inf_{y \in \Delta^{\mathcal{I}}} R^{\ T}(x,y) \Rightarrow C^{\ T}(y) \} \end{array} $
Assertions: Inclusion axioms:	$\langle a:C, n \rangle, \mathcal{I} \models \langle a:C, n \rangle$ iff C individual <i>a</i> is instance	$\mathcal{L}^{\mathcal{I}}(a^{\mathcal{I}}) \ge n$ (similarless of concept <i>C</i> at lease of concept <i>C</i> at lease	y for r ast to	oles) degree <i>n</i> , $n \in [0, 1] \cap \mathbb{Q}$

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Fuzzy DL: Specific Constructs

- Concrete data types
 - e.g., Sedan \sqcap (\geqslant price 22.000)
- Fuzzy constraints
 - numerical features may be constrained by so-called fuzzy membership functions

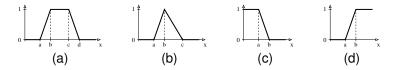


Figure: (a) Trapezoidal function trz(a, b, c, d), (b) triangular function tri(a, b, c), (c) left shoulder function ls(a, b), and (d) right shoulder function rs(a, b).

► For instance, *item*4's price is about 24000

item4: 3price.tri(22000, 24000, 26000)

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Definition (Specific Concept Expressions)

 $\begin{array}{lll} C & \rightarrow & \forall t.d \mid \exists t.d \ (\mathsf{fuzzy \ constraints}) \\ d & \rightarrow & \mathit{ls}(a,b) \mid \mathit{rs}(a,b) \mid \mathit{tri}(a,b,c) \mid \mathit{trz}(a,b,c,d) \end{array}$

e.g.

Car ⊓ ∃*price*. *tri*(22000, 24000, 26000)

$$\begin{array}{rcl} C & \to & DR & (datatype restriction) \\ DR & \to & (\geqslant t \ val) \ | \ (\leqslant t \ val) \ | \ (= t \ val) \\ val & \to & string \ | \ rational \ | \ FN \ | \ AE \\ FN & \to & rational \ | \ fuzzynumber \ | \ FN_1 \ \star \ FN_2 \ \ \star \in \{+, -, \cdot, \div\} \\ AE & \to & rational \ | \ t \ | \ n \cdot t \ | \ AE_1 + AE_2 \end{array}$$

e.g.

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e.g.,

 $\textit{NiceHotel} \doteq 0.3 \cdot \textit{CheapHotel} + 0.7 \cdot \textit{ConfortableHotel}$

 $C \rightarrow mod(C)$ (modified concept)

where mod is a linear hedge. E.g.,

SportCar ⊑ Car ⊓ ∃hasSpeed.very(High)

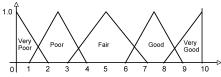
Example

Assume that we have to chose among three offers for a GIS system that have been evaluated according to

Criteria: Cost, Delivery Time and Quality

Assume the decision matrix and the definition of the vague performance values are

	Offer	Cost 0.258	DeliveryTime 0.105	Quality 0.637		
Ì	a ₁	VeryPoor	Fair	Good		
	a ₂	Good	VeryGood	Poor		
	a ₃	Fair	Fair	Poor		



Fuzzy DL encoding:

 $VeryPoor \doteq Is(0, 2), Poor \doteq tri(1, 2.5, 4), Fair \doteq tri(3, 5, 7), Good \doteq tri(6, 7.5, 9), VeryGood \doteq rs(8, 10)$

a1 :Alternative □ ∃hasCost.VeryPoor □ ∃hasDeliveryTime.Fair □ ∃hasQuality.Good

a₂:Alternative □ ∃hasCost.Good □ ∃hasDeliveryTime.VeryGood □ ∃hasQuality.Poor

a₃:Alternative □ ∃hasCost.Fair □ ∃hasDeliveryTime.Fair □ ∃hasQuality.Poor

Alternative = (= hasRankValue 0.258 · hasCost + 0.105 · hasDeliveryTime + 0.637 · hasQuality)

Final Rank Value: rank(K, a_i) = mom(K, Alternative, a_i, hasRankValue)

$rank(\mathcal{K}, a_1)$	=	5.301
$rank(\mathcal{K}, a_2)$	=	4.577
$rank(\mathcal{K}, a_3)$	=	3.408

$$a^* = \arg \max_{a_i} \operatorname{rank}(\mathcal{K}, a_i) = a_1$$

- Encoding nicely extends if background knowledge is involved such as, *e.g.*,
 - Criteria taxonomy

Consistency

DataQualityElement

Properties of alternatives, *e.g.*,

a₁:Alternative ⊓ ∃hasSecurity.VeryPoor

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Conclusions & Outlook

- We have made a first attempt towards MCDM within fuzzy DLs, *i.e.*,
 - Towards a (fuzzy) knowledge-assisted approach to decision making
- FUZZYDL reasoner supports the encoding proposed here
- The MCDM literature (inclusive their fuzzy MCDM variants) is quite large
- It will be of interest to look at
 - how to integrate and support different MCDM methods
 - a methodology to smoothly integrate background knowledge into MCDM
 - whether (fuzzy) knowldege/ontology-based MCDM is an added-value in real-world applications

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