Multi Criteria Decision Making in Fuzzy Description Logics: A First Step

Umberto Straccia

ISTI-CNR, Pisa, Italy

straccia@isti.cnr.it

www.straccia.info

Introduction

- In the last years the interest in ontologies has significantly grown
- An ontology is defined as an explicit and formal specification of a shared conceptualization
- Description Logics (DLs) are a family of logics that are the logical foundation of the standard W3C ontology language OWL [HPS04].

- It is widely agreed that “classical” ontology languages are not appropriate to deal with fuzzy/vague knowledge
- Fuzzy ontologies emerge as useful in several applications, such as multimedia information retrieval, image interpretation, ontology mapping, matchmaking and the Semantic Web [LS08]
- Several fuzzy extensions of DLs can be found in the literature (see the survey in [LS08])
- Some fuzzy DL reasoners have been implemented, such as FUZZYDL [BS08], DELOREAN [BDGR08] or FIRE [SSSK06].

- In this work, we make a first step in combining Multi-Criteria Decision Making (MCDM) and fuzzy DLs
  - $\Rightarrow$ fuzzy knowledge assisted approach to decision making

Preliminaries: Mathematical Fuzzy Logic [Háj98]

- Fuzzy statements: $\langle \phi, n \rangle$, where $n \in [0,1]$ and ϕ is a statement
  - The degree of truth of ϕ is *at least n*
- Fuzzy interpretation: $\mathcal{I} : Atoms \rightarrow [0,1]$ and is then extended inductively:

$$
\begin{array}{ll}
\mathcal{I}(\phi \wedge \psi) = \mathcal{I}(\phi) \otimes \mathcal{I}(\psi) & \mathcal{I}(\phi \vee \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi), \\
\mathcal{I}(\phi \rightarrow \psi) = \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi) & \mathcal{I}(\neg \phi) = \ominus \mathcal{I}(\phi)\,, \\
\mathcal{I}(\exists x.\phi(x)) = \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c)) & \mathcal{I}(\forall x.\phi(x)) = \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c))
\end{array}
$$

$\otimes$, $\oplus$, $\Rightarrow$, and $\ominus$ are *truth combination functions*

	Łukasiewicz Logic	Gödel Logic	Product Logic	"Zadeh Logic"
$a \otimes b$	$\max(a+b-1, 0)$	$\min(a, b)$	$a \cdot b$	$\min(a, b)$
$a \oplus b$	$\min(a+b, 1)$	$\max(a, b)$	$a + b - a \cdot b$	$\max(a, b)$
$a \Rightarrow b$	$\min(1 - a + b, 1)$	$\begin{cases} 1 & \text{if } a \leqslant b \\ b & \text{otherwise} \end{cases}$	$\min(1, b/a)$	$\max(1 - a, b)$
$\ominus a$	$1 - a$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$1 - a$

- $\mathcal{I} \models \langle \phi, n \rangle$ iff $\mathcal{I}(\phi) \geqslant n$
- Best Entailment Degree (BED): $bed(\mathcal{K}, \phi) = \sup \{ r \mid \mathcal{K} \models \langle \phi, r \rangle \}$
- BED can be computed as (where $\phi \leqslant x$ is $\langle \neg\phi, 1 - x \rangle$)

$$bed(\mathcal{K}, \phi) = \min x. \text{ such that } \mathcal{K} \cup \{\phi \leqslant x\} \text{ satisfiable}$$

- E.g., for Łukasiewicz logic, we may use Mixed Integer Linear Programming

$$\begin{aligned} bed(\mathcal{K}, \phi) = \min x. \text{ such that } & \\ & x \in [0,1], x_{\neg\phi} \geqslant 1 - x, \sigma(\neg\phi), \\ & \text{for all } \langle \phi', n \rangle \in \mathcal{K}, x_{\phi'} \geqslant n, \sigma(\phi'), \end{aligned}$$

$$\sigma(\phi) = \begin{cases} x_p \in [0,1] & \text{if} \quad \phi = p \\ x_{\phi'} = \ominus x_\phi, x_\phi \in [0,1] & \text{if} \quad \phi = \neg\phi' \\ \begin{array}{l} x_{\phi_1} \otimes x_{\phi_2} = x_\phi, \\ \sigma(\phi_1), \sigma(\phi_2), x_\phi \in [0,1] \end{array} & \text{if} \quad \phi = \phi_1 \wedge \phi_2 \\ x_{\phi_1} \oplus x_{\phi_2} = x_\phi & \text{if} \quad \phi = \phi_1 \vee \phi_2 \\ \sigma(\neg\phi_1 \vee \phi_2) & \text{if} \quad \phi = \phi_1 \rightarrow \phi_2 \,. \end{cases}$$

Preliminaries: MCDM Basics

- Alternatives A_i: different choices of action available to the decision maker to be ranked
- Decision criteria C_j: different dimensions from which the alternatives can be viewed and evaluated
- Decision weights w_j: importance of a criteria
- Performance weights a_{ij}: performance of alternative w.r.t. a decision criteria

		Criteria				
		w_1	w_2	.	.	w_m
Alternatives		C_1	C_2	.	.	C_m
x_1	A_1	a_{11}	a_{12}	.	.	a_{1m}
x_2	A_2	a_{21}	a_{22}	.	.	a_{2m}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
x_n	A_n	a_{n1}	a_{n2}	.	.	a_{nm}

(1)

- Final ranking value x_i:

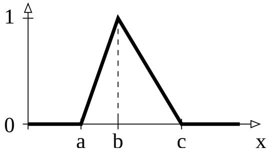
$$x_i = \sum_{j=1}^{m} a_{ij} w_j$$

- Optimal alternative A^*:

$$A^* = \arg\max_{A_i} x_i$$

Preliminaries: Fuzzy MCDM Basics

- Principal difference: weights w_i and performance a_{ij} are fuzzy numbers
- Fuzzy number $\tilde{n}$: fuzzy set over relas with triangular membership function $tri(a, b, c)$. Intended being an approximation of the number b



- Any real value n is seen as the fuzzy number $tri(n, n, n)$
- Arithmetic operators $+, -, \cdot$ and $\div$ are extended to fuzzy numbers
  - For $* \in \{+, \cdot\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * a_2, b_1 * b_2, c_1 * c_2)$
  - For $* \in \{-, \div\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * c_2, b_1 * b_2, c_1 * a_2)$
- Final ranking value x_i: fuzzy number

$$\tilde{x}_i = \sum_{j=1}^{m} \tilde{a}_{ij} \cdot \tilde{w}_j$$

- Optimal alternative A^*:

$$A^* = \arg\max_{A_i} x_i^{defuzzy}$$

using some defuzzification method for fuzzy numbers

Towards MCDM in Fuzzy Description Logic

- Our extension of to fuzzy DLs is grounded on the fuzzy DL $\mathcal{ALCF}(D)$ [Str05]
- We will just provide a minimal variant of $\mathcal{ALCF}(D)$ to deal with MCDM
- Recall that fuzzy $\mathcal{ALCF}(D)$ is the basic DL $\mathcal{ALC}$ extended with functional roles (letter $\mathcal{F}$) and concrete domains [LM07] (letter D) allowing to deal with data types such as strings, integers, reals and fuzzy membership functions

Description Logics (DLs)

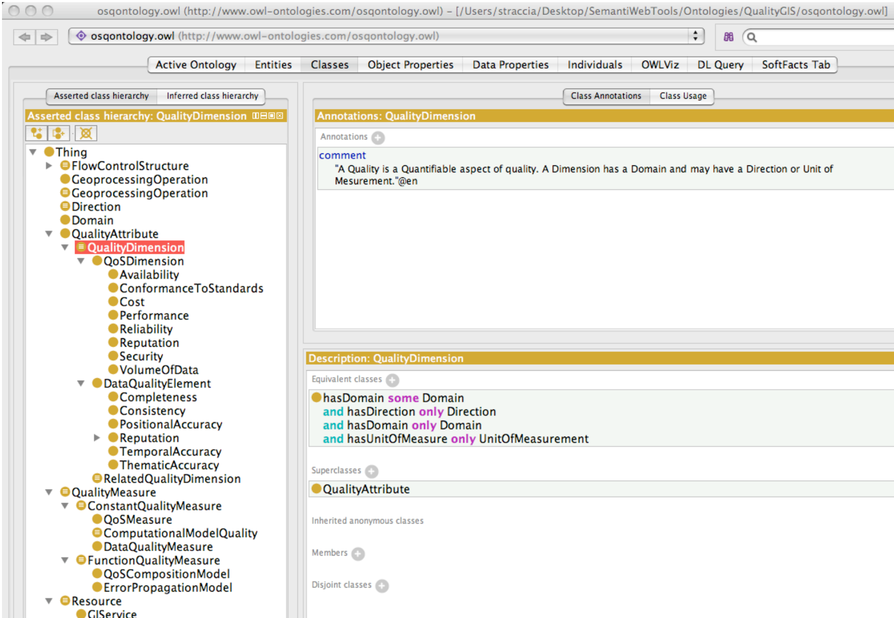
- The logics behind OWL-DL and OWL-Lite, `http://dl.kr.org/`.
- Concept/Class: names are equivalent to unary predicates
  - In general, concepts equiv to formulae with one free variable
- Role or attribute: names are equivalent to binary predicates
  - In general, roles equiv to formulae with two free variables
- Taxonomy: Concept and role hierarchies can be expressed
- Individual: names are equivalent to constants
- Operators: restricted so that:
  - Language is decidable and, if possible, of low complexity
  - No need for explicit use of variables
    - Restricted form of $\exists$ and $\forall$
  - Features such as counting can be succinctly expressed

The Crisp DL Family

- A given DL is defined by set of concept and role forming operators
- Basic language: $\mathcal{ALC}$($\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplement)

Syntax				Semantics	Example
C, D	$\rightarrow$	$\top$	|	$\top(x)$	
		$\bot$	|	$\bot(x)$	
		A	|	$A(x)$	$Human$
		$C \sqcap D$	|	$C(x) \land D(x)$	$Human \sqcap Male$
		$C \sqcup D$	|	$C(x) \lor D(x)$	$Nice \sqcup Rich$
		$\neg C$	|	$\neg C(x)$	$\neg Meat$
		$\exists R.C$	|	$\exists y.R(x, y) \land C(y)$	$\exists has\_child.Blond$
		$\forall R.C$		$\forall y.R(x, y) \rightarrow C(y)$	$\forall has\_child.Human$
$C \sqsubseteq D$				$\forall x.C(x) \rightarrow D(x)$	$Happy\_Father \sqsubseteq Man \sqcap \exists has\_child.Female$
$a{:}C$				$C(a)$	$John{:}Happy\_Father$

Example: GIS Quality Assessment Ontology [OWML08]



Note on DL Naming

$\mathcal{AL}$: $C, D \longrightarrow \top \mid \bot \mid A \mid C \sqcap D \mid \neg A \mid \exists R.\top \mid \forall R.C$

$\mathcal{C}$: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + \mathcal{C}$

$\mathcal{S}$: Used for $\mathcal{ALC}$ with transitive roles $\mathcal{R}_+$

$\mathcal{U}$: Concept disjunction, $C_1 \sqcup C_2$

$\mathcal{E}$: Existential quantification, $\exists R.C$

$\mathcal{H}$: Role inclusion axioms, $R_1 \sqsubseteq R_2$, *e.g.*, *is_component_of* $\sqsubseteq$ *is_part_of*

$\mathcal{N}$: Number restrictions, $(\geqslant n\ R)$ and $(\leqslant n\ R)$, *e.g.*, $(\geqslant 3\ has\_Child)$ (has at least 3 children)

$\mathcal{Q}$: Qualified number restrictions, $(\geqslant n\ R.C)$ and $(\leqslant n\ R.C)$, *e.g.*, $(\leqslant 2\ has\_Child.Adult)$ (has at most 2 adult children)

$\mathcal{O}$: Nominals (singleton class), $\{a\}$, *e.g.*, $\exists has\_child.\{mary\}$.
Note: $a{:}C$ equiv to $\{a\} \sqsubseteq C$ and $(a, b){:}R$ equiv to $\{a\} \sqsubseteq \exists R.\{b\}$

$\mathcal{I}$: Inverse role, R^-, *e.g.*, $isPartOf = hasPart^-$

$\mathcal{F}$: Functional role, f, *e.g.*, $functional(hasAge)$

$\mathcal{R}_+$: transitive role, *e.g.*, $transitive(isPartOf)$

$\mathcal{R}$: role inclusions with composition, $R_1 \circ R_2 \sqsubseteq S$, *e.g.*, $isPartOf \circ isPartOf \sqsubseteq isPartOf$

For instance,

$$\begin{array}{lcll}
\mathcal{SHIF} & = & \mathcal{S}+\mathcal{H}+\mathcal{I}+\mathcal{F} = \mathcal{ALCR}_+\mathcal{HIF} & \text{OWL-Lite} \\
\mathcal{SHOIN} & = & \mathcal{S}+\mathcal{H}+\mathcal{O}+\mathcal{I}+\mathcal{N} = \mathcal{ALCR}_+\mathcal{HOIN} & \text{OWL-DL} \\
\mathcal{SROIQ} & = & \mathcal{S}+\mathcal{R}+\mathcal{O}+\mathcal{I}+\mathcal{Q} = \mathcal{ALCR}_+\mathcal{ROIN} & \text{OWL 2}
\end{array}$$

Fuzzy DLs Basics

The semantics is an immediate consequence of applying mathematical fuzzy logic to the First-Order-Logic translation of DLs expressions

Interpretation:

$$\mathcal{I} = \Delta^{\mathcal{I}} \qquad C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1] \qquad R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$$

$\otimes$ = t-norm
$\oplus$ = s-norm
$\ominus$ = negation
$\Rightarrow$ = implication

Concepts:

Syntax			Semantics		
C, D	$\longrightarrow$	$\top \mid$	$\top^{\mathcal{I}}(x)$	$=$	1
		$\bot \mid$	$\bot^{\mathcal{I}}(x)$	$=$	0
		$A \mid$	$A^{\mathcal{I}}(x)$	$\in$	$[0,1]$
		$C \sqcap D \mid$	$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	${C_1}^{\mathcal{I}}(x) \otimes {C_2}^{\mathcal{I}}(x)$
		$C \sqcup D \mid$	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	${C_1}^{\mathcal{I}}(x) \oplus {C_2}^{\mathcal{I}}(x)$
		$\neg C \mid$	$(\neg C)^{\mathcal{I}}(x)$	$=$	$\ominus C^{\mathcal{I}}(x)$
		$\exists R.C \mid$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y)$
		$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(u)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \Rightarrow C^{\mathcal{I}}(y)\}$

Assertions: $\langle a{:}C, n\rangle$, $\mathcal{I} \models \langle a{:}C, n\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geqslant n$ (similarly for roles)

- individual a is instance of concept C at least to degree n, $n \in [0,1] \cap \mathbb{Q}$

Inclusion axioms: $\langle C \sqsubseteq D, n\rangle$,

- $\mathcal{I} \models \langle C \sqsubseteq D, n\rangle$ iff $\inf_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x) \geqslant n$

Fuzzy DL: Specific Constructs

- Concrete data types
  - *e.g.*, $Sedan \sqcap (\geqslant price\ 22.000)$
- Fuzzy constraints
  - numerical features may be constrained by so-called fuzzy membership functions

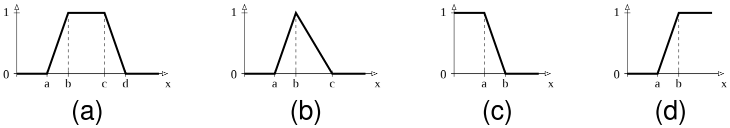


Figure: (a) Trapezoidal function $trz(a, b, c, d)$, (b) triangular function $tri(a, b, c)$, (c) left shoulder function $ls(a, b)$, and (d) right shoulder function $rs(a, b)$.

- For instance, *item*4's price is about 24000

$$item4 : \exists price.tri(22000, 24000, 26000)$$

Definition (Specific Concept Expressions)

$$\begin{array}{lcl} C & \rightarrow & \forall t.d \mid \exists t.d \quad \text{(fuzzy constraints)} \\ d & \rightarrow & ls(a,b) \mid rs(a,b) \mid tri(a,b,c) \mid trz(a,b,c,d) \end{array}$$

e.g.

$$Car \sqcap \exists price.tri(22000, 24000, 26000)$$

$$\begin{array}{lcl} C & \rightarrow & DR \quad \text{(datatype restriction)} \\ DR & \rightarrow & (\geqslant t\ val) \mid (\leqslant t\ val) \mid (= t\ val) \\ val & \rightarrow & string \mid rational \mid FN \mid AE \\ FN & \rightarrow & rational \mid fuzzynumber \mid FN_1 \star FN_2 \quad \star \in \{+, -, \cdot, \div\} \\ AE & \rightarrow & rational \mid t \mid n \cdot t \mid AE_1 + AE_2 \end{array}$$

e.g.

$$audi234 : Sedan \sqcap (\leqslant price\ 26000)$$

$$\begin{array}{l} SoldItem \sqsubseteq (= totalPrice\ netprice + VAT) \\ SoldItem \sqsubseteq (= VAT\ 0.2 \cdot netprice) \end{array}$$

$$\begin{array}{lcl} C & \rightarrow & WC \quad \text{(weighted sum concept)} \\ WC & \rightarrow & (w_1 \cdot C_1 + w_2 \cdot C_2 + \ldots + w_k \cdot C_k) \end{array}$$

e.g.,

$$NiceHotel \doteq 0.3 \cdot CheapHotel + 0.7 \cdot ConfortableHotel$$

$$C \rightarrow mod(C) \quad \text{(modified concept)}$$

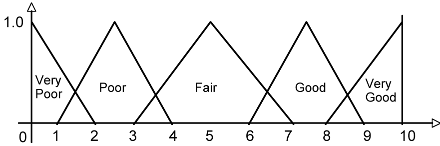
where *mod* is a linear hedge. E.g.,

$$SportCar \sqsubseteq Car \sqcap \exists \text{hasSpeed}.very(High)$$

Example

- Assume that we have to chose among three offers for a GIS system that have been evaluated according to
  - Criteria: Cost, Delivery Time and Quality
- Assume the decision matrix and the definition of the vague performance values are

Offer	Cost 0.258	DeliveryTime 0.105	Quality 0.637
a_1	VeryPoor	Fair	Good
a_2	Good	VeryGood	Poor
a_3	Fair	Fair	Poor



- Fuzzy DL encoding:

$$\text{VeryPoor} \doteq \text{ls}(0,2), \quad \text{Poor} \doteq \text{tri}(1, 2.5, 4), \quad \text{Fair} \doteq \text{tri}(3,5,7), \quad \text{Good} \doteq \text{tri}(6, 7.5, 9), \quad \text{VeryGood} \doteq \text{rs}(8, 10)$$

$$\begin{array}{l} \text{a}_1 : \text{Alternative} \sqcap \exists \text{hasCost.VeryPoor} \sqcap \exists \text{hasDeliveryTime.Fair} \sqcap \exists \text{hasQuality.Good} \\ \text{a}_2 : \text{Alternative} \sqcap \exists \text{hasCost.Good} \sqcap \exists \text{hasDeliveryTime.VeryGood} \sqcap \exists \text{hasQuality.Poor} \\ \text{a}_3 : \text{Alternative} \sqcap \exists \text{hasCost.Fair} \sqcap \exists \text{hasDeliveryTime.Fair} \sqcap \exists \text{hasQuality.Poor} \end{array}$$

$$\text{Alternative} \doteq (= \text{hasRankValue } 0.258 \cdot \text{hasCost} + 0.105 \cdot \text{hasDeliveryTime} + 0.637 \cdot \text{hasQuality})$$

- Final Rank Value: $\text{rank}(\mathcal{K}, \text{a}_i) = \textit{mom}(\mathcal{K}, \text{Alternative}, \text{a}_i, \text{hasRankValue})$

$$\begin{array}{rcl} \text{rank}(\mathcal{K}, \text{a}_1) & = & 5.301 \\ \text{rank}(\mathcal{K}, \text{a}_2) & = & 4.577 \\ \text{rank}(\mathcal{K}, \text{a}_3) & = & 3.408 \end{array}$$

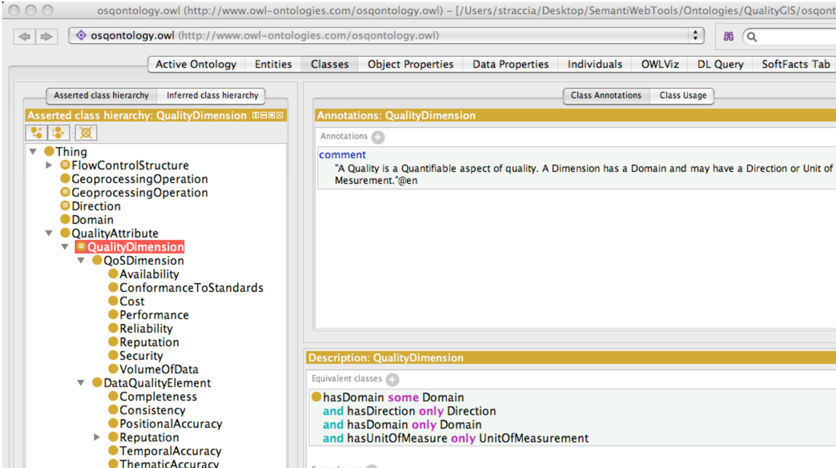
$$\text{a}^* = \arg\max_{\text{a}_i} \text{rank}(\mathcal{K}, \text{a}_i) = \text{a}_1$$

- Encoding nicely extends if background knowledge is involved such as, *e.g.*,
  - Criteria taxonomy

$$\text{Consistency} \sqsubseteq \text{DataQualityElement}$$

  - Properties of alternatives, *e.g.*,

$$a_1 : \text{Alternative} \sqcap \exists \text{hasSecurity.VeryPoor}$$



Conclusions & Outlook

- We have made a first attempt towards MCDM within fuzzy DLs, *i.e.*,
  - Towards a (fuzzy) knowledge-assisted approach to decision making
- FUZZYDL reasoner supports the encoding proposed here
- The MCDM literature (inclusive their fuzzy MCDM variants) is quite large
- It will be of interest to look at
  - how to integrate and support different MCDM methods
  - a methodology to smoothly integrate background knowledge into MCDM
  - whether (fuzzy) knowldege/ontology-based MCDM is an added-value in real-world applications

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